Entanglement transitions in monitored Fermonic chains

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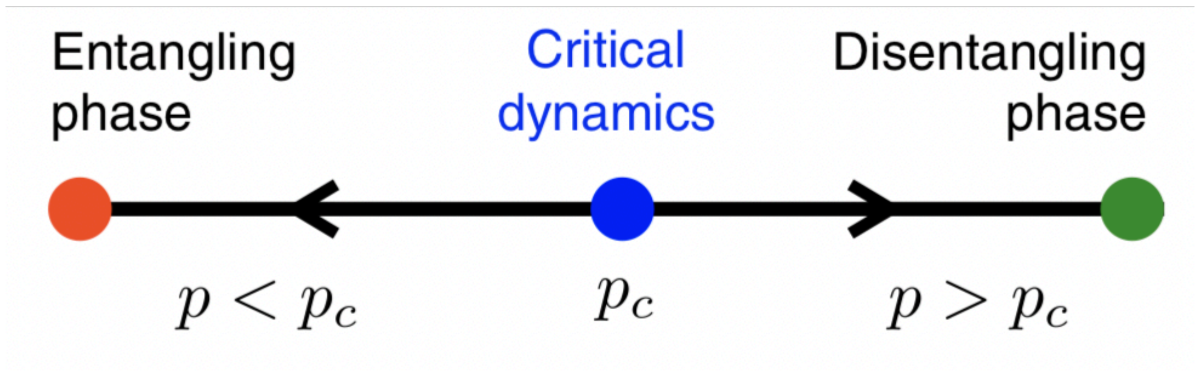
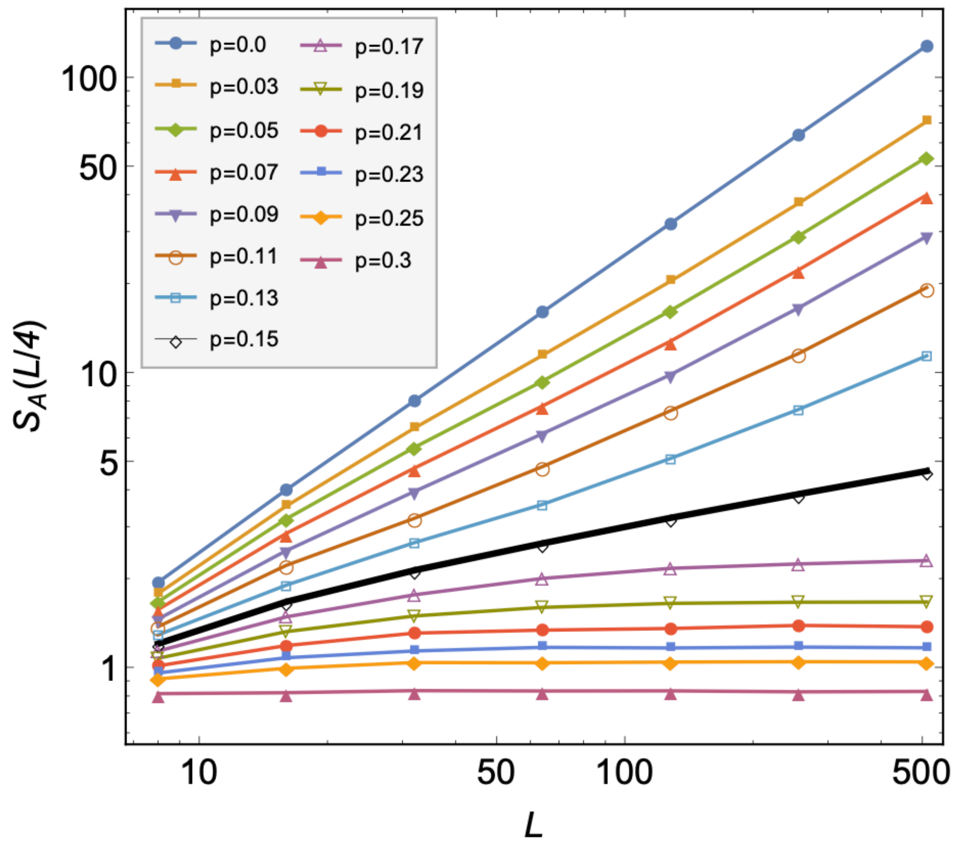
Outline

- **Open quantum systems & quantum measurements**
  → unraveling the master equation

- **Quantum "resilience" to monitoring**

- **Stability of entanglement transitions in Fermionic systems:**
  → role of the different unravelings
  → dissipators with variable range
  → Hamiltonian interactions





Entanglement transitions ?

The quantum measurement process

Take a ***quantum system*** coupled to an ***external environment***
(weak coupling, Born-markov & secular approximation)

- The system is fully quantum
- The environment is a "*<u>classical measurement apparatus</u>*"

$\boxed{\text{System monitoring originates a } \textbf{stochastic quantum dynamics}}$

M. B. Plenio, P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998)

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Quantum trajectory: $|\psi_\xi(t)\rangle$
specific sequence of measurements & outcomes
→ single realization of the **stochastic process** $\xi(t)$

$$d|\psi\rangle = |u\rangle\, dt + |\nu\rangle\, d\xi(t)$$

stochastic evolution equation

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stochastic evolution equation

Trajectories **do not interfere** with each other

$$\rho(t) = \overline{|\psi_\xi(t)\rangle\langle\psi_\xi(t)|}$$

averaging provides a Lindblad-type evolution

$$\dot{\rho}(t) = -\mathrm{i}\big[H, \rho(t)\big] - \frac{\gamma}{2} \sum_j \left(L_j^\dagger L_j \rho + \rho L_j^\dagger L_j - 2 L_j \rho L_j^\dagger \right)$$

(mean effect of the stochastic process)

M. B. Plenio, P. L. Knight, Rev. Mod. Phys. **70**, 101 (1998)

Unraveling the master equation

Many different unravelings $\{\,|\psi_\xi(t)\rangle\}$ for the same state $\rho(t) = \sum_\xi p_\xi\,|\psi_\xi(t)\rangle\langle\psi_\xi(t)\,|$

Wiseman and Milburn, *Quantum Measurement and Control* (Cambridge Univ. Press, 2009)
Barchielli and Gregoratti, *Quantum trajectories and measurements in continuous time* (Springer 2009)

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The system may be:

A) continuously measured

$$d\,|\psi\rangle = \left[-\,\mathrm{i}H - \frac{\gamma}{2}\sum_j \left(L_j - \langle L_j\rangle\right)^2\right]|\psi\rangle\,dt + \sqrt{\gamma}\sum_j \left(L_j - \langle L_j\rangle\right)|\psi\rangle\,\boxed{d\xi_j}$$

$$\left(L_j = L_j^\dagger\right)$$

$d\xi_j$ **normally distributed variables** with <u>zero mean</u> and <u>variance $\gamma\,dt$.</u>

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stochastic Schrödinger equation

$$|\psi_\xi(t+\delta t)\rangle \sim e^{\left[\sum_j \delta\xi_j + (2\langle L_j\rangle - 1)\,\gamma\,\delta t\right] L_j}\, e^{-\mathrm{i}\,H\,\delta t}\,|\psi_\xi(t)\rangle$$

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The system may be:

B) subject to occasional, yet abrupt, measuring

$$d\,|\psi\rangle = \left[-\,\mathrm{i}H_{\mathrm{eff}} - \frac{\gamma}{2}\sum_j \langle L_j^2\rangle\right]|\psi\rangle\,dt + \sum_j \left[\frac{L_j}{\sqrt{\langle L_j^2\rangle}} - 1\right]|\psi\rangle\,\boxed{d\xi_j}$$

$$\left(L_j = L_j^\dagger\right)$$

$d\xi_j = 0$, except at random times (j-th jump) when it becomes 1:

$$d\xi_i\,d\xi_j = \delta_{ij}\,d\xi_j\,, \qquad \langle d\xi_j\rangle = \gamma\,\langle L_j^2\rangle\,dt$$

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non-Hermitian Schrödinger equation + quantum jumps

$$H_{\mathrm{eff}} = H - \mathrm{i}\frac{\gamma}{2}\sum_j L_j^2 \qquad |\psi(t)\rangle \to \begin{cases} L_j|\psi(t)\rangle & [\,\delta p_j = \langle\delta\xi_j\rangle\,] \\ e^{-\mathrm{i}\,H_{\mathrm{eff}}\,\delta t}|\psi(t)\rangle & [p = 1 - \sum_j \delta p_j\,] \end{cases}$$

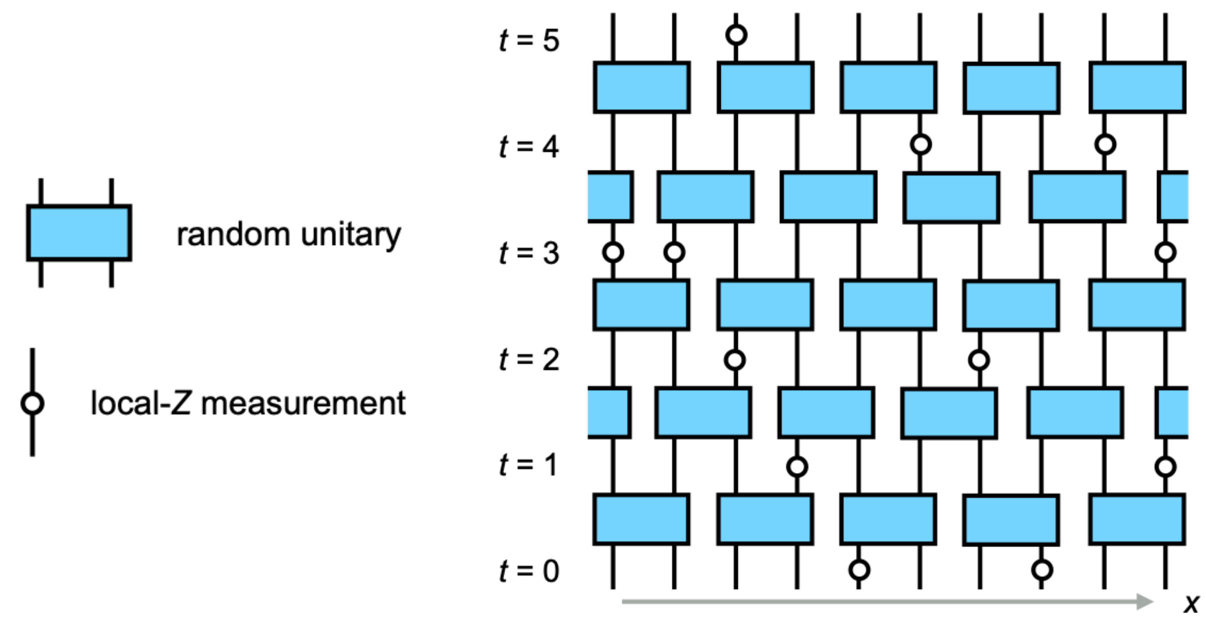
Wiseman and Milburn, *Quantum Measurement and Control* (Cambridge Univ. Press, 2009)
Barchielli and Gregoratti, *Quantum trajectories and measurements in continuous time* (Springer 2009)

Measurement-induced entanglement transitions

Recently growing theoretical focus

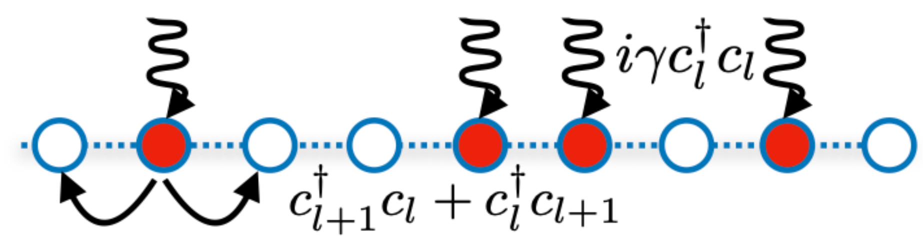
starting from *random circuits*

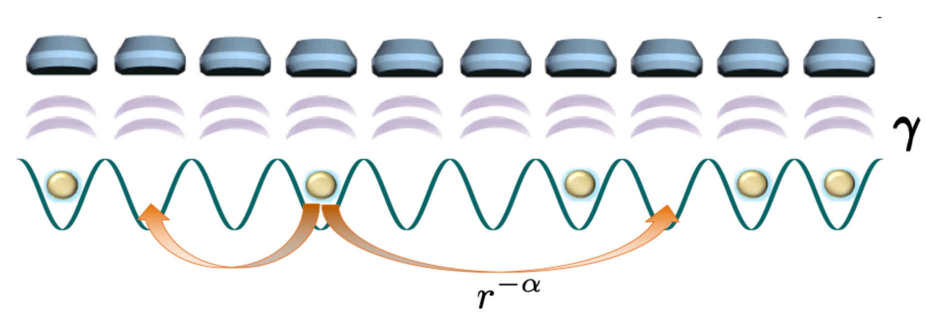
Li, Chen Fisher, (PRB 2018, PRB 2019)
Skinner, Ruhman, Nahum (PRX 2019)
Chan, Nandkishore, Pretko, Smith (PRB 2019)
Szyniszewski, Romito, Schomerus (PRB 2019, PRL 2020)
Gullans, Huse (PRX 2020, PRL 2020)
Choi, Bao, Qi, Altman (PRL 2020, PRB 2021)
Nahum, Roy, Skinner, Ruhman (PRX Quantum 2021)
Jian, You, Vasseur, Ludwig (PRB 2020)
Medina, Vasseur, Serbyn (PRB 2021)
Zabalo, Gullans, Wilson, Gopalakrishnan, Huse, Pixley (PRB 2020, PRL 2022)
…



and then also for *Hamiltonian systems*

Cao, Tilloy, De Luca (SciPost Phys. 2019)
Goto, Danshita (PRA 2020)
Tang, Zhu (PRRes 2020)
Lang, Büchler (PRB 2020)
Alberton, Buchhold, Diehl (PRL 2021)
Botzung, Diehl, Müller (PRB 2021)
Lunt, Pal (PRR 2020, PRB 2021)
Minato, Sugimoto, Kuwahara, Saito (PRL 2022)
Coppola, Tirrito, Karevski, Collura (PRB 2022)
Tirrito, Santini, Fazio, Collura (arXiv 2022)
Turkheshi, Biella, Fazio, Dalmonte, Schirò (PRB 2020 & 2021 & 2022)
Turkheshi, Schirò (arXiv 2022)
…





Free-Fermion Hamiltonian

$$H = -\sum_j \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j + \mathrm{Hc}\right) + h\sum_j \left(2n_j - 1\right)$$

(mappable into an Ising chain, after Jordan-Wigner transform)

* Unitary dynamics ruled by a quench in $h: \ +\infty \to h_f$
* Two different unravelings of the same Lindblad master equation:
  1) QSD with $L_j = n_j$
  2) QT with $L_j = 1 + \alpha n_j \quad (\alpha = 1)$

G. Piccitto, A. Russomanno, DR, Phys. Rev. B **105**, 064305 (2022)

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<u>Gaussianity is preserved</u>: $|\psi\rangle = \mathcal{N} \exp\left(\frac{1}{2}\sum_{j,l=1}^{L} Z_{jl}\, c_j^\dagger c_l^\dagger\right)|0\rangle$

quadratic antisymmetric form

by applying exponentials of operators quadratic in $c_j^{(\dagger)}$

$$\left[\, n_j = \frac{e^{x n_j} - 1}{e^x - 1} \quad \longrightarrow \quad L_j = 1 + \alpha n_j = e^{n_j \log(1+\alpha)} \,\right]$$

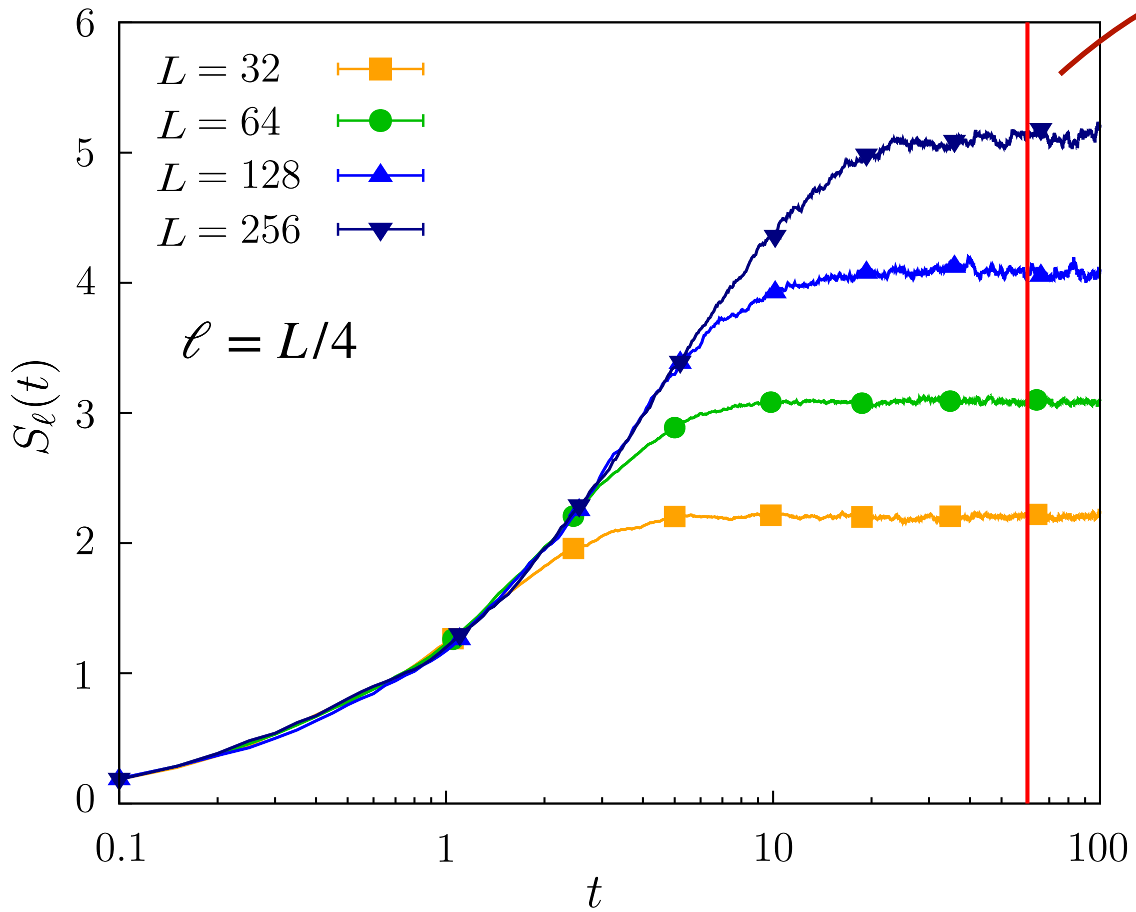
$$[\, x = \log(1+\alpha)\,]$$

G. Piccitto, A. Russomanno, DR, Phys. Rev. B **105**, 064305 (2022)

Results for QSD

genuine quantum correlations

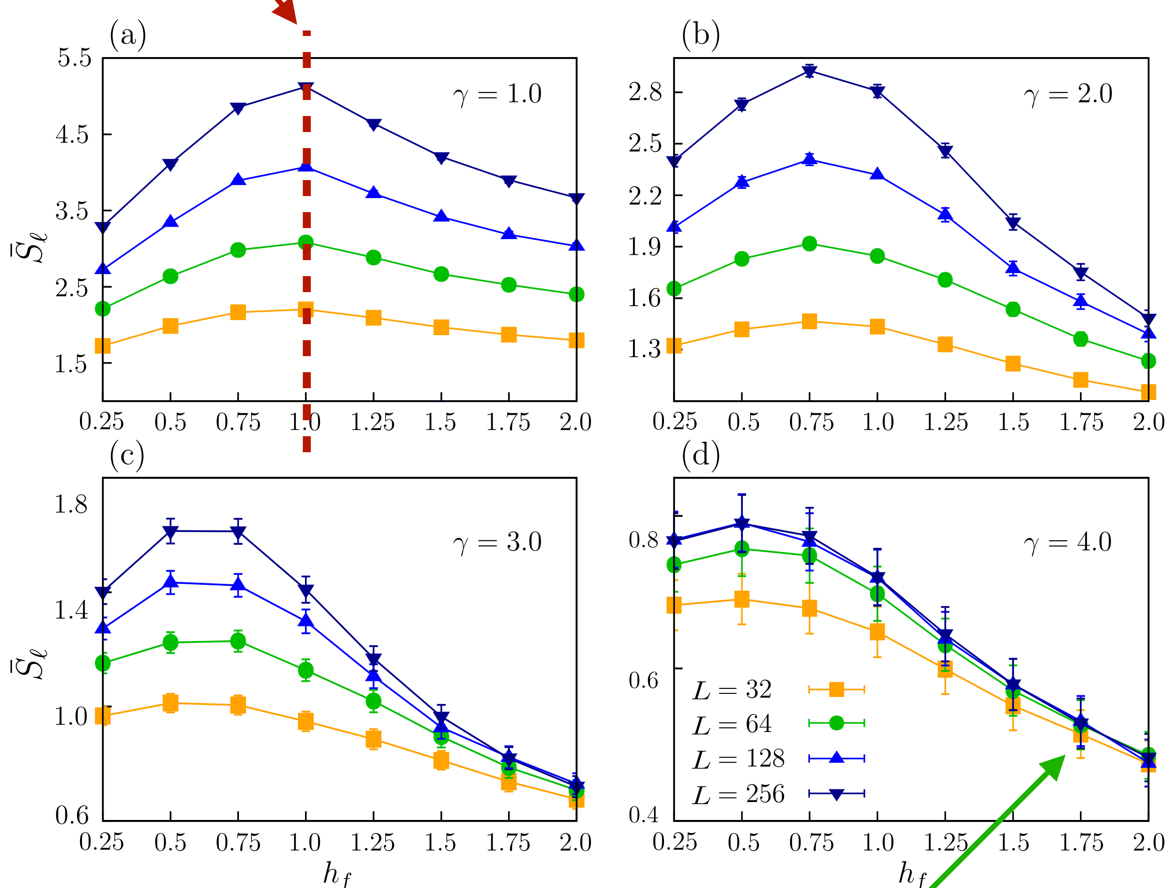
$$S_\ell(t) \equiv \overline{S_\ell(|\psi_\xi(t)\rangle\langle\psi_\xi(t)|)}$$



$$\overline{S_\ell(|\psi_\xi(t)\rangle\langle\psi_\xi(t)|)} \le S_\ell(\overline{|\psi_\xi(t)\rangle\langle\psi_\xi(t)|})$$

quantum ≤ quantum+classical

subextensive behavior $S_\ell \sim \log L$

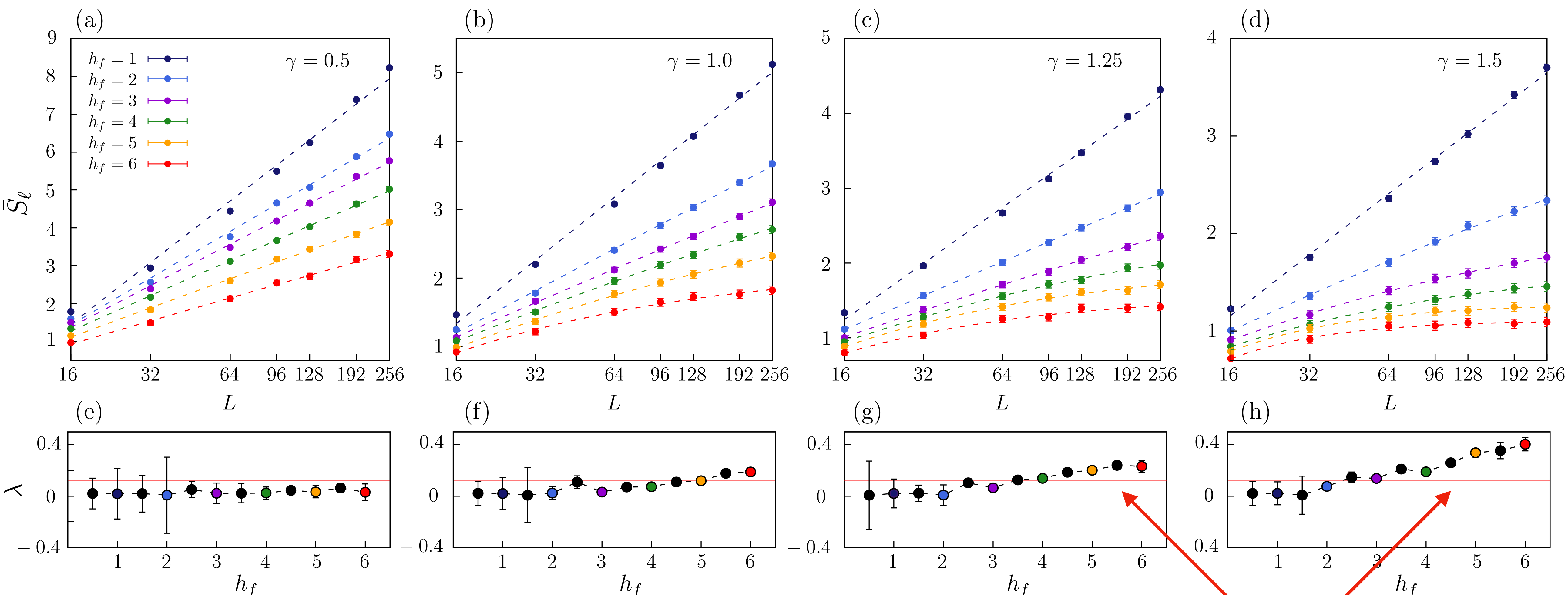


area-law behavior $S_\ell \sim \text{cost}$

Onset of a measurement-driven entanglement transition?

Results for QSD: finite-size study

Fitting numerical data with $\boxed{f(L) \sim \tanh\left[\lambda(h_f,\gamma)\,\log L\right]}$

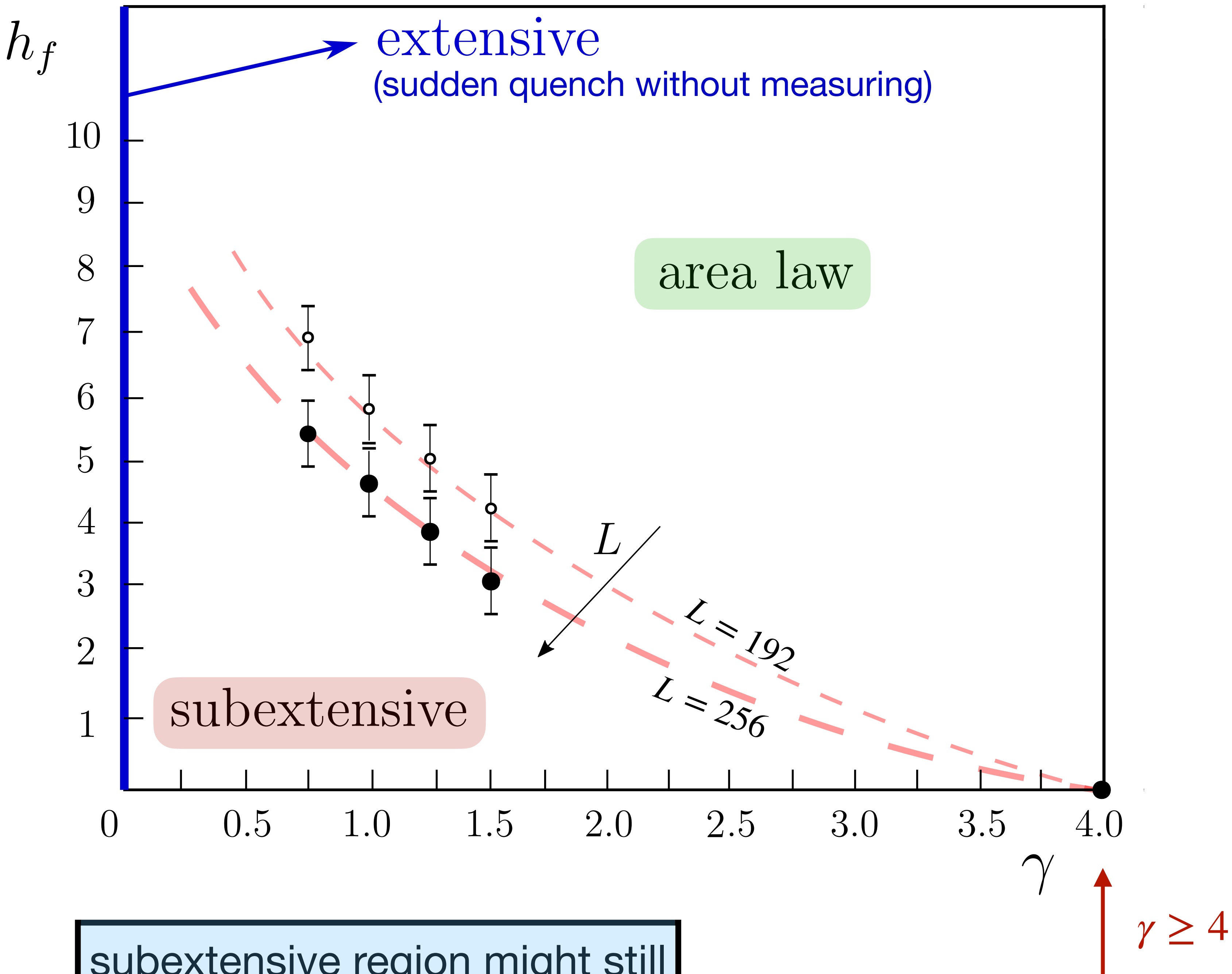


$\lambda = \left(\log L_{\max}\right)^{-1}$

$\lambda \log(L_{\max}) \geq 1$: Reliable fit $\Rightarrow$ entanglement will likely behave **area-law**

$\lambda \log(L_{\max}) \ll 1$: Unreliable fit $\Rightarrow$ entanglement will likely behave **log-law**

Results for QSD: nonequilibrium phase diagram



subextensive region might still be a finite-size crossover

X. Turkeshi, A. Biella, R. Fazio, M. Dalmonte, M. Schirò
Phys. Rev. B **103**, 224210 (2021)

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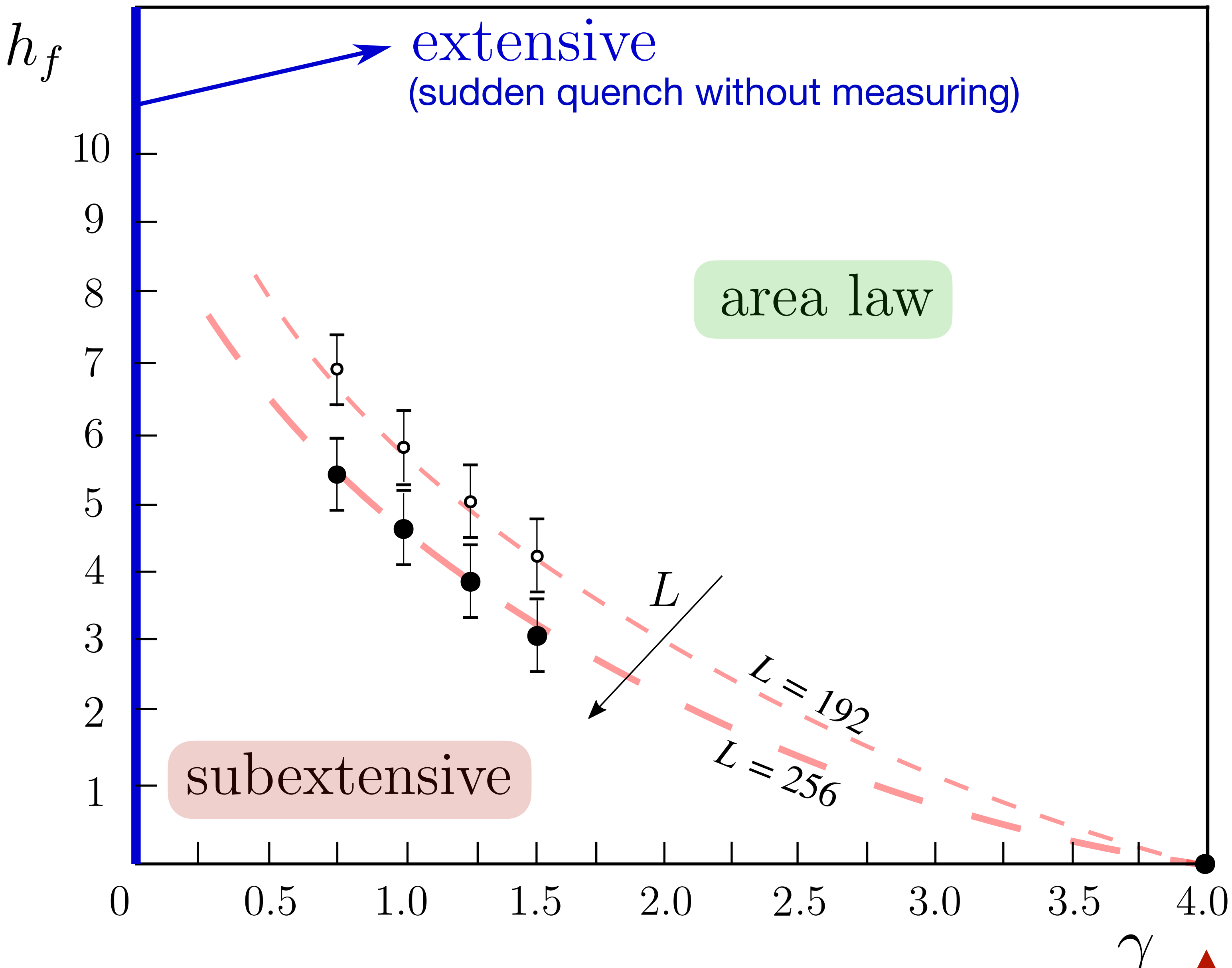
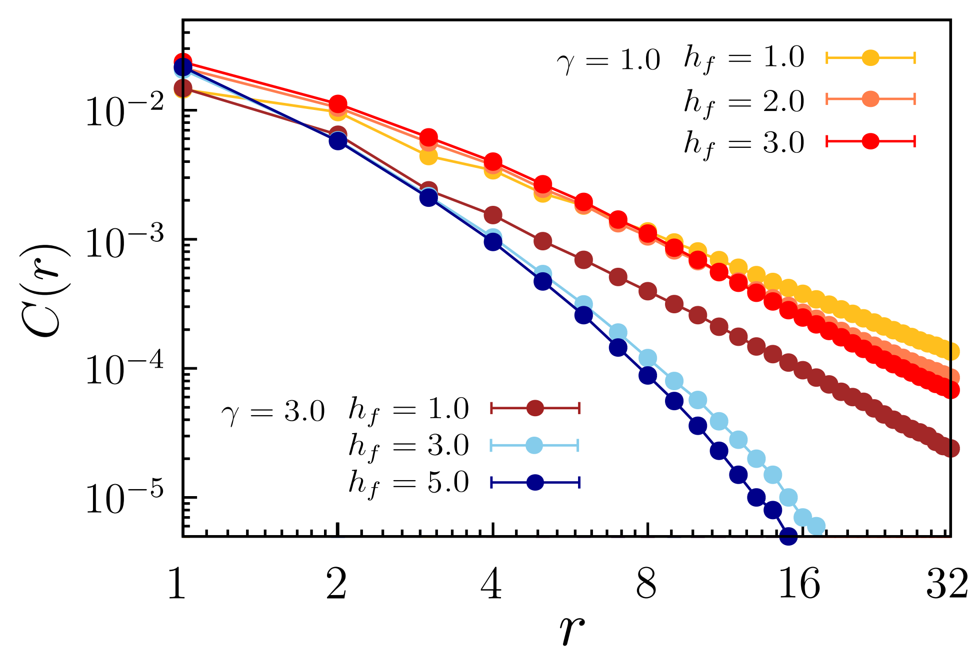


$\gamma \geq 4$

$$C_j(t,r) = \overline{|\langle\psi_\xi(t)|c_j^\dagger c_{j+r}|\psi_\xi(t)\rangle|^2}$$



subextensivity → power-law decay
area-law → exponential decay

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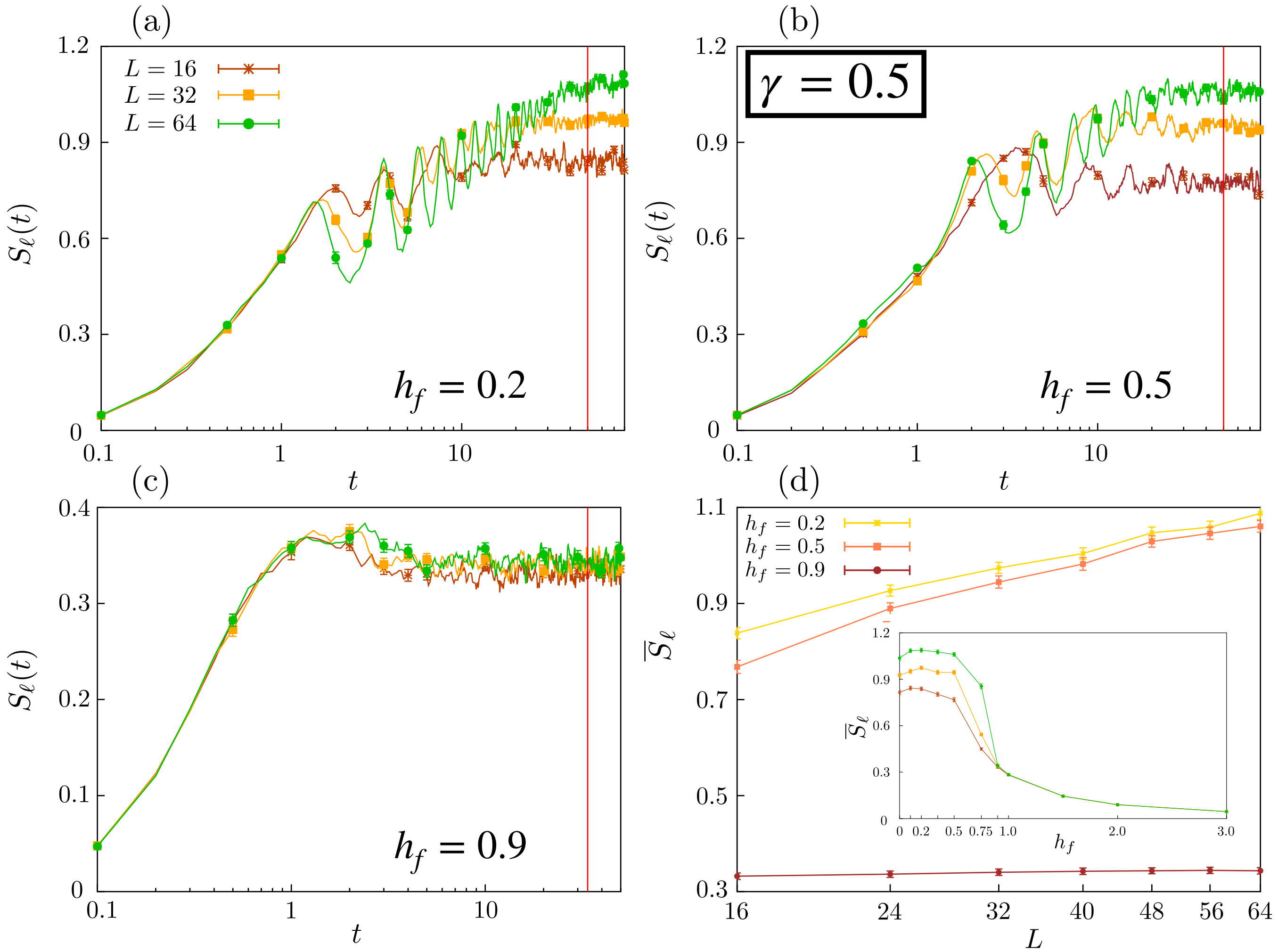
Results for QJ

Lindblad jump operators: $L_j = 1 + n_j$

(same unraveling as QSD with $L_j = n_j$)



see also: X. Turkeshi, M. Dalmonte, R. Fazio, M. Schirò, Phys. Rev. B **105**, L241114 (2022)

Entanglement entropy seems to undergo a change log-law → area-law at **much smaller values of** h_f

is subextensivity a finite-size crossover?

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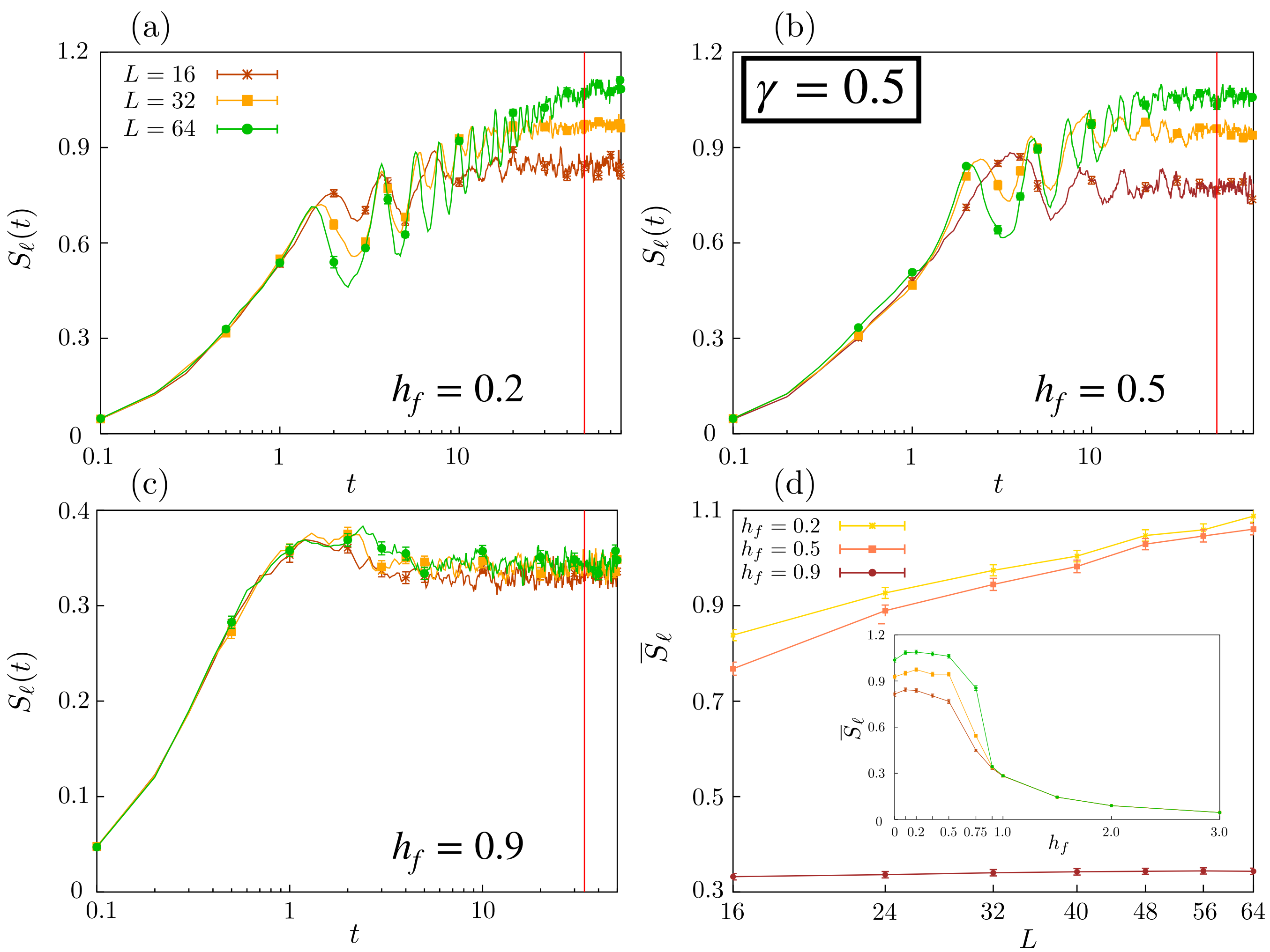

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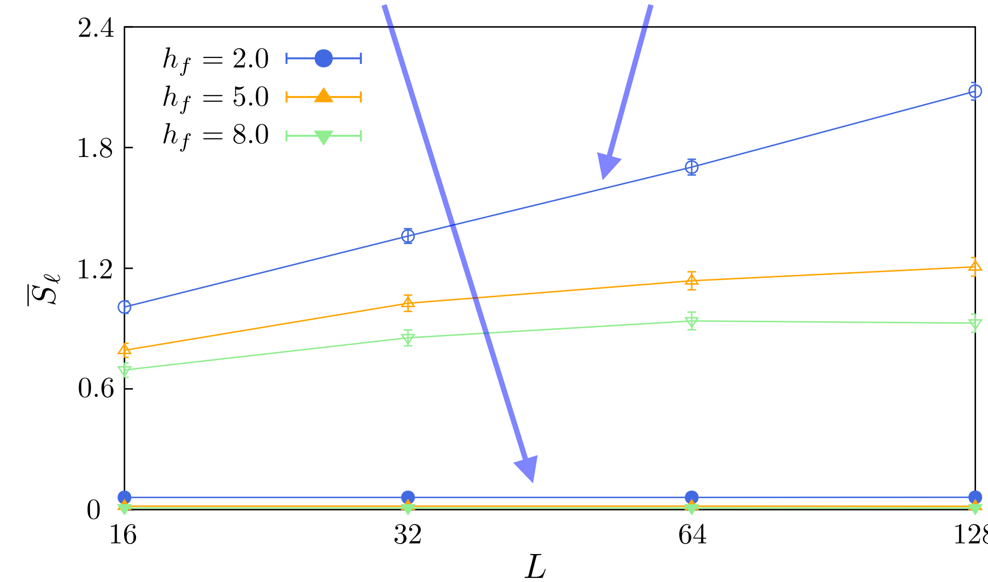
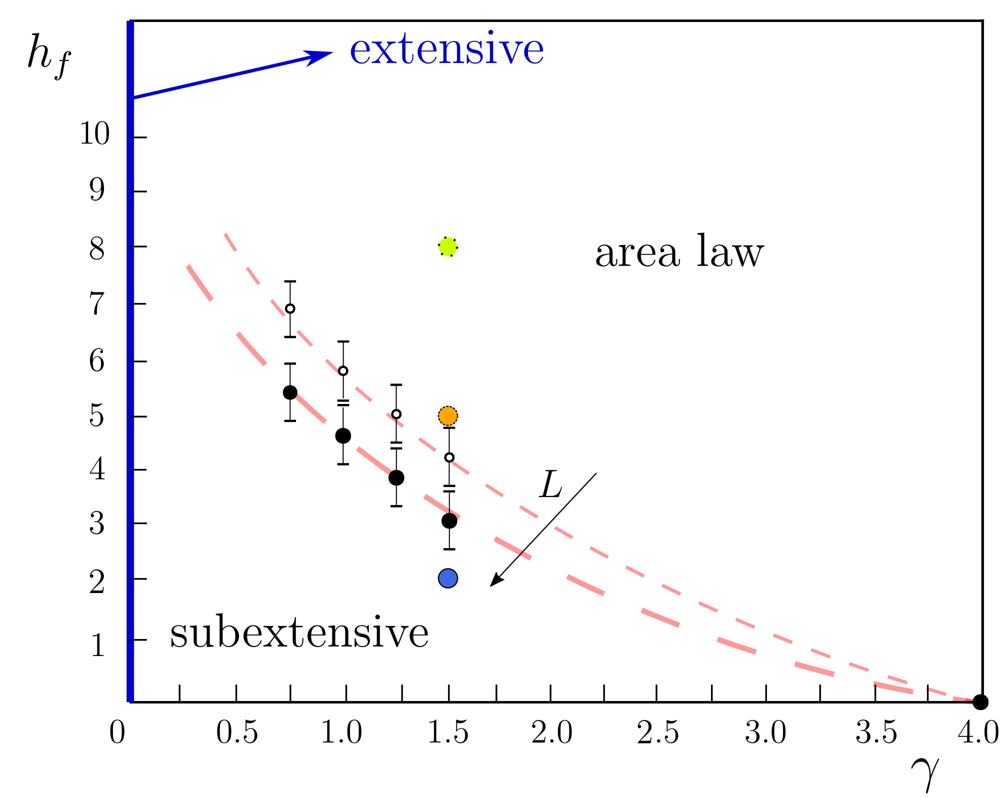
Different unravelings provide **different entanglement behaviors**:

QT **vs.** **QSD**





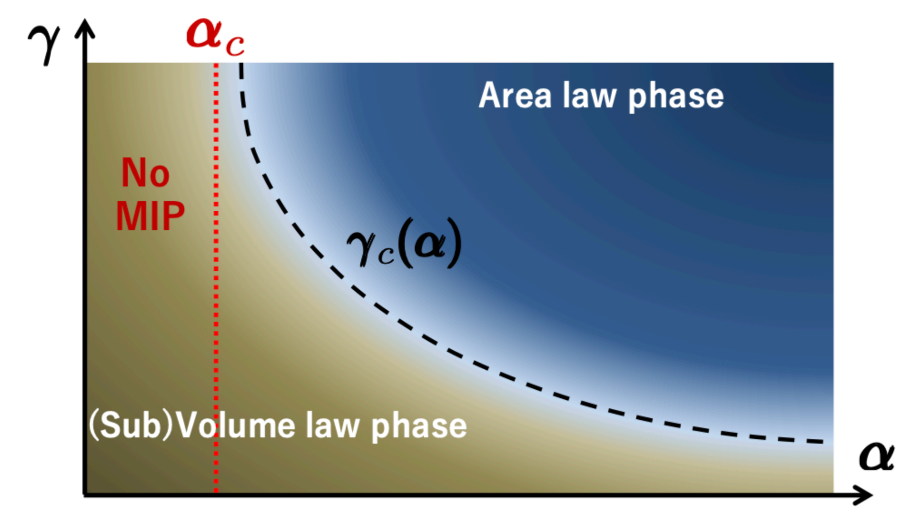
Free-Fermions & long-range dissipators

$$H = -\sum_j \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j + \mathrm{Hc}\right) + h \sum_j \left(2n_j - 1\right)$$

(mappable into an Ising chain, after Jordan-Wigner transform)

Long-range Hamiltonian interactions are expected to modify the nonequilibrium entanglement phase diagram

T. Minato, K. Sugimoto, T. Kuwahara, K. Saito, PRL **128**, 010603 (2022)
M. Block, Y. Bao, S. Choi, E. Altman, N. Y. Yao, PRL **128**, 010604 (2022)



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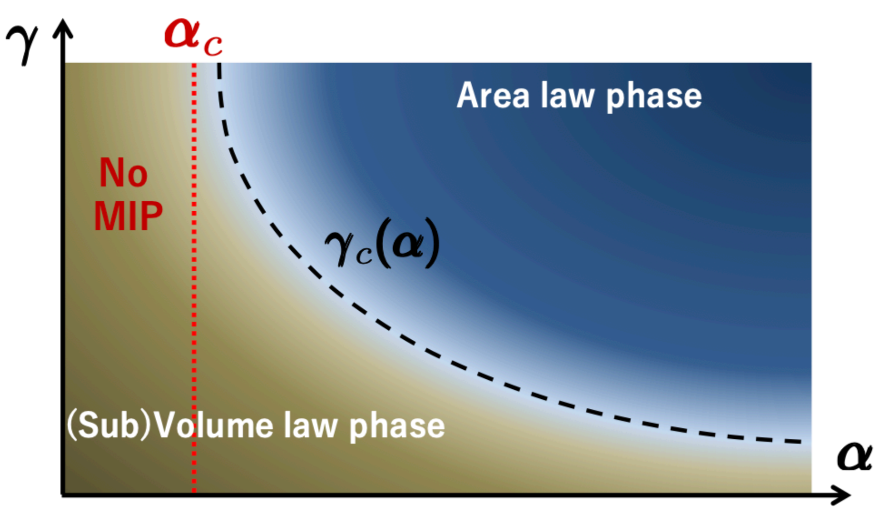


⇨ We analyze continuous measurements through the QSD unraveling

$$L_i = \sum_j \frac{1}{|i-j|^\alpha}\, \mathrm{i}\left(c_i - c_i^\dagger\right)\left(c_j + c_j^\dagger\right)$$ long-range Hermitian dissipators

<u>Gaussianity is still preserved, also in this case</u>

G. Piccitto, A. Russomanno, DR, in preparation

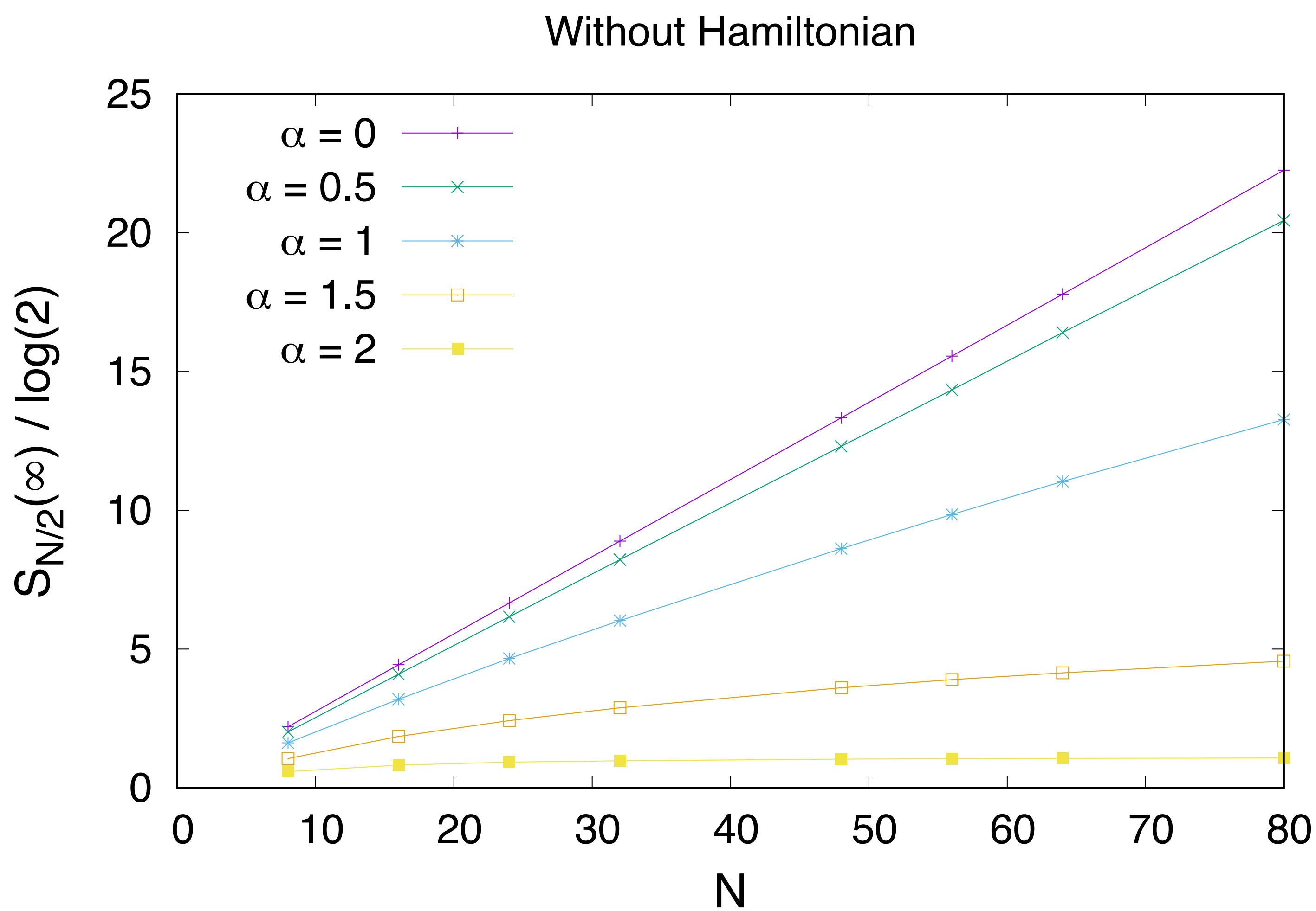
QSD unraveling with long-range dissipators

A) Only long-range dissipators

$\gamma = 1$

$$L_i = \sum_j \frac{1}{|i-j|^\alpha}\, \mathrm{i}\left(c_i - c_i^\dagger\right)\left(c_j + c_j^\dagger\right)$$

A direct transition from **volume-law** to **area-law** behavior for the entanglement entropy seems to emerge:



QSD unraveling with long-range dissipators

B) Long-range dissipators

$\gamma = 1$

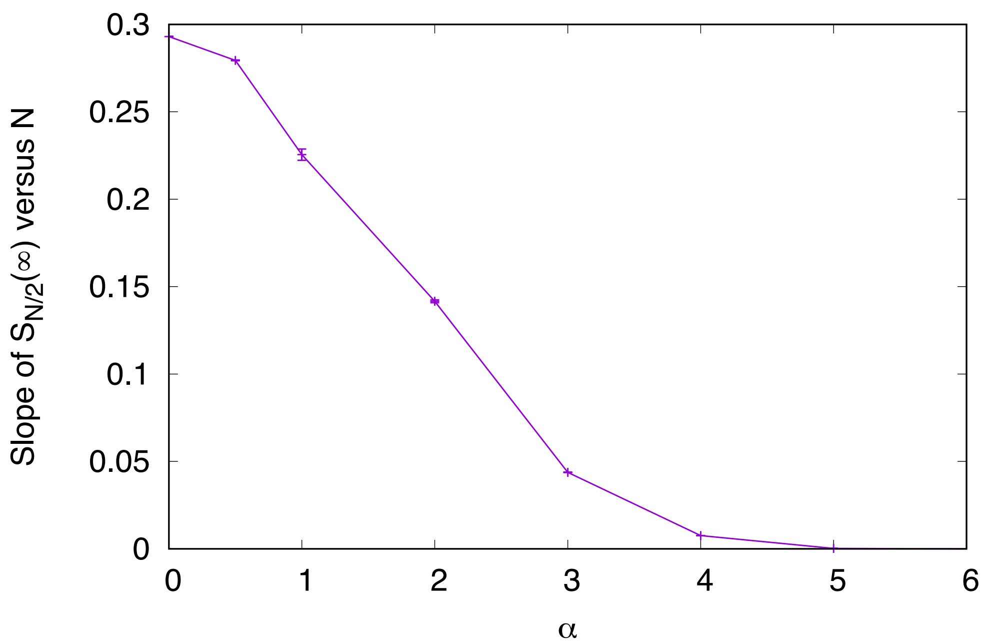
$$L_i = \sum_j \frac{1}{|i-j|^{\alpha}} \mathrm{i}\left(c_i - c_i^{\dagger}\right)\left(c_j + c_j^{\dagger}\right)$$

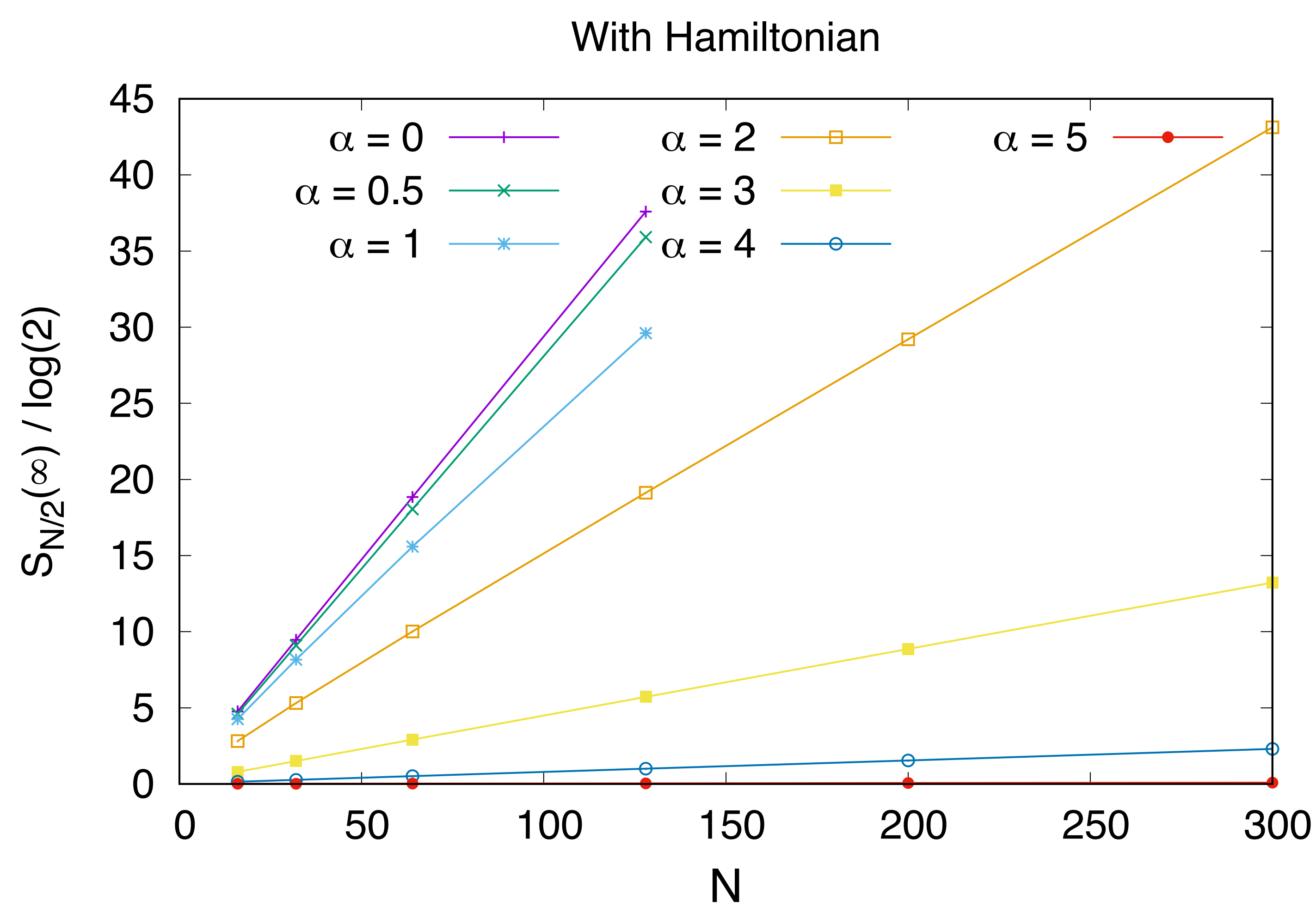
+ Hamiltonian

$h: +\infty \to 0.5$

$$H = -\sum_j \left(c_j^{\dagger} c_{j+1} + c_{j+1}^{\dagger} c_j + \mathrm{Hc}\right) + h \sum_j \left(2n_j - 1\right)$$

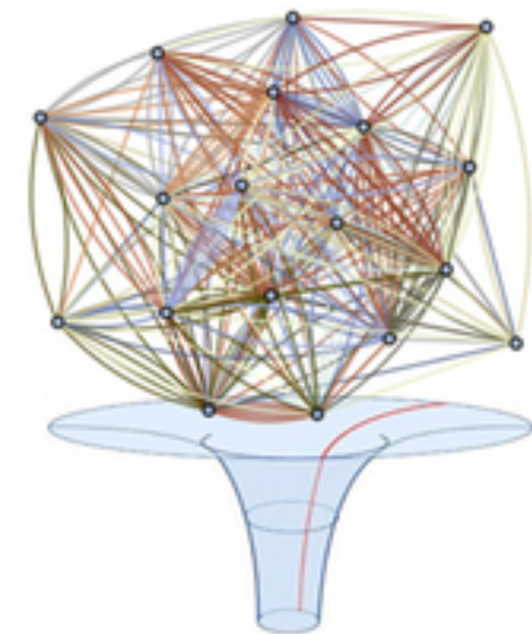
volume-law to **area-law**
entanglement transition





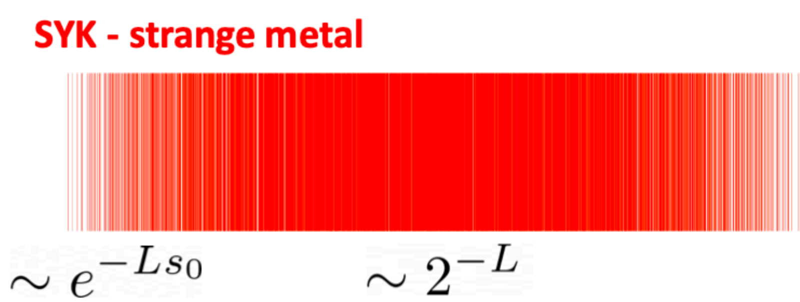
Heavily interacting fermions

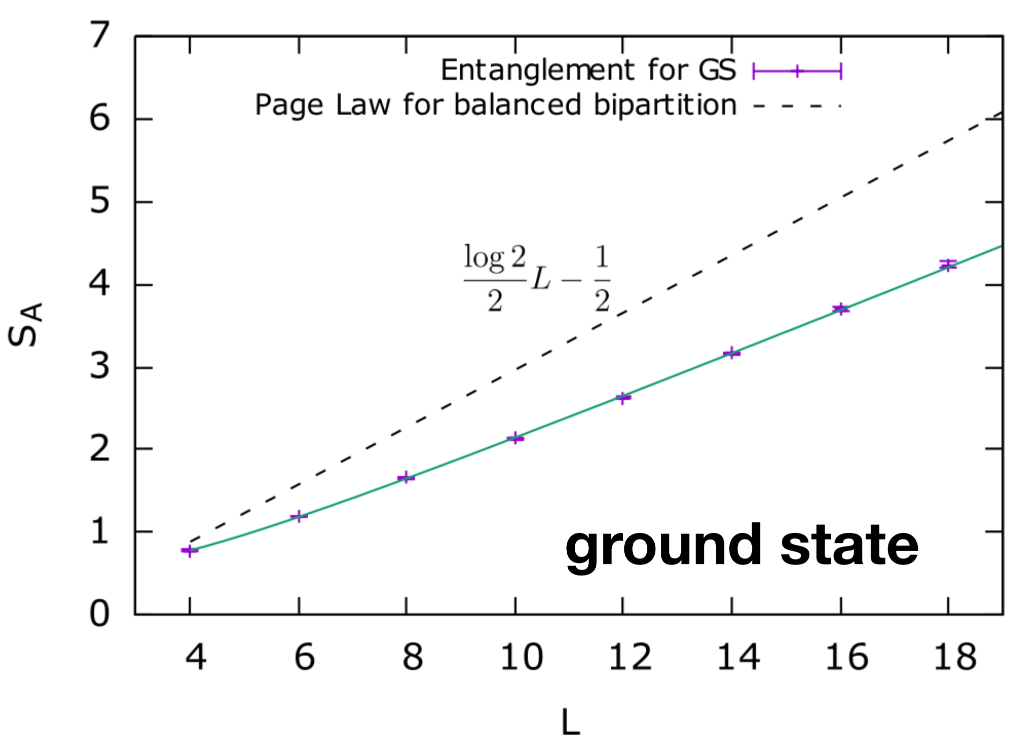
$$H = \sum_{i,j,k,l} \boxed{J_{ij;kl}}\, c_i^\dagger c_j^\dagger c_k c_l$$ Sachdev-Ye-Kitaev (SYK) model



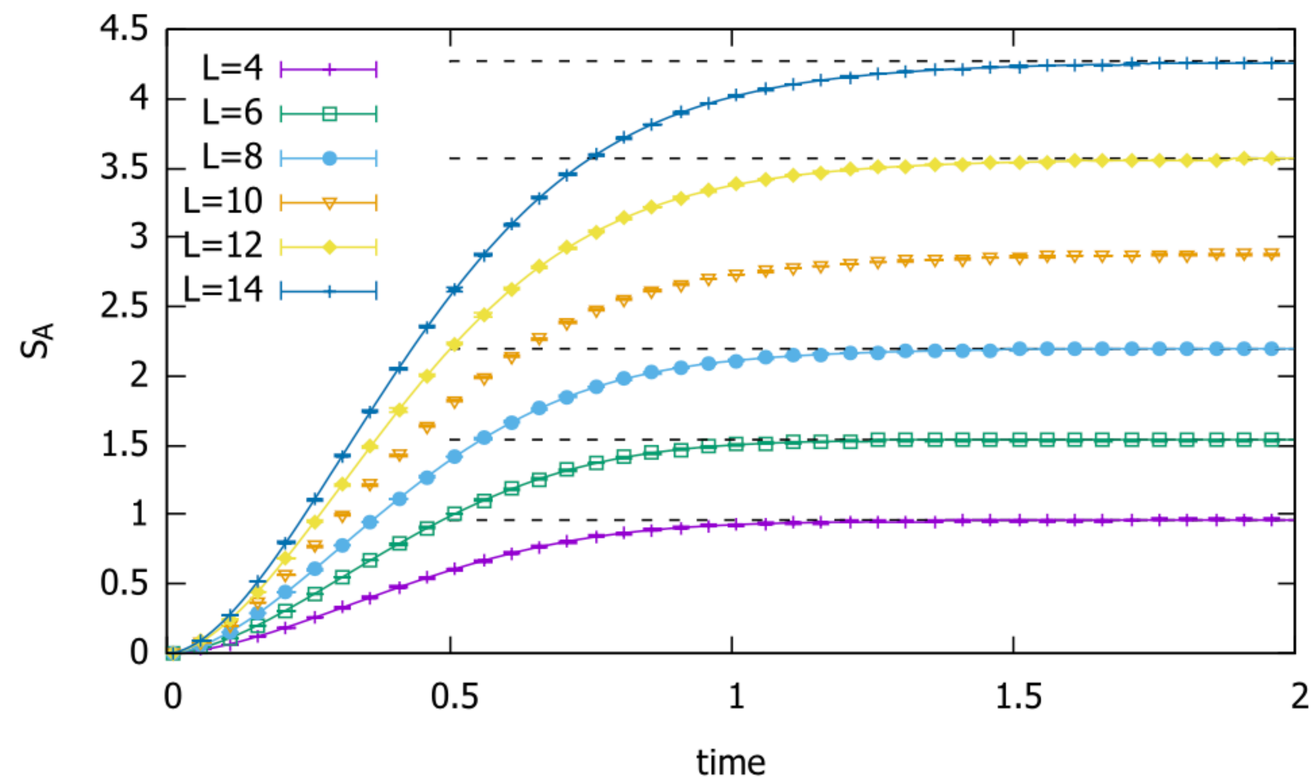
Random Gaussian couplings: $\overline{J_{ij;kl}} = 0 \quad \overline{|J_{ij;kl}|^2} = J^2/L^3$

A non-integrable, highly chaotic paradigm



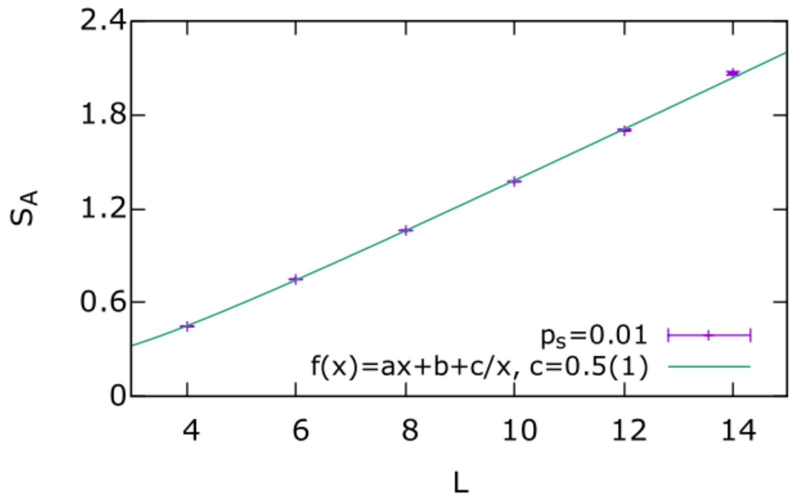


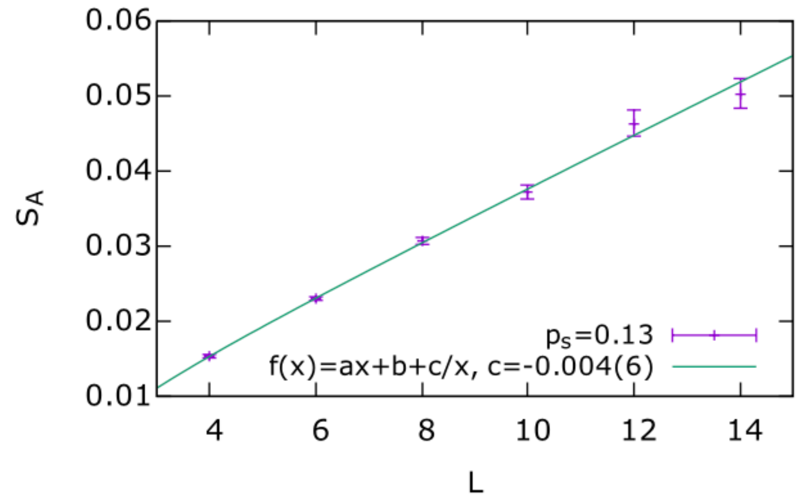
Unitary dynamics (starting from a product state)



SYK model - projective measurements

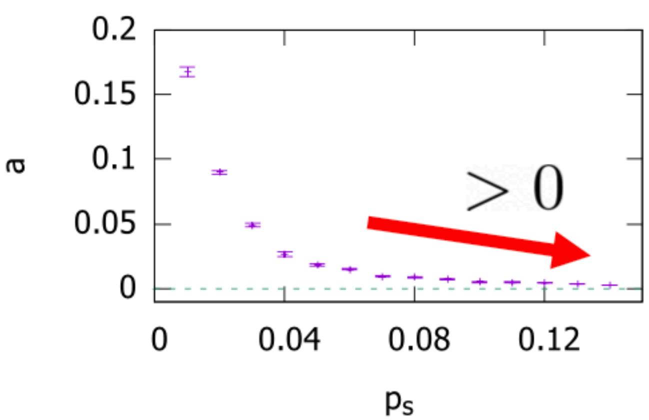
We go in the Poissonian limit: $dt_{PM} \to 0, \quad p_t \to 0, \quad dt_{PM}/p_t = \text{cost}$

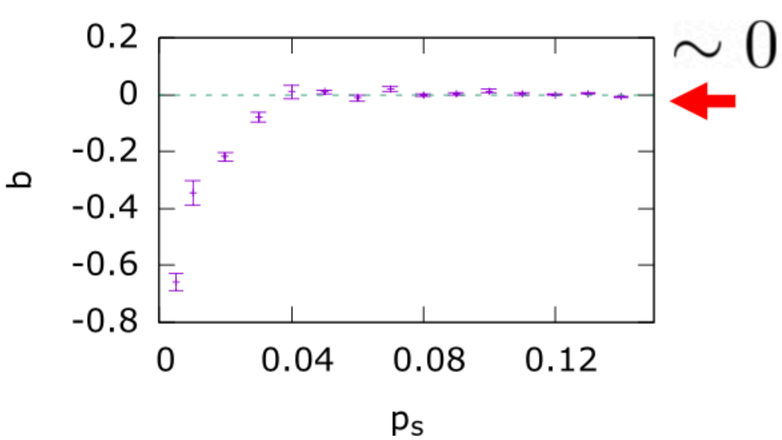


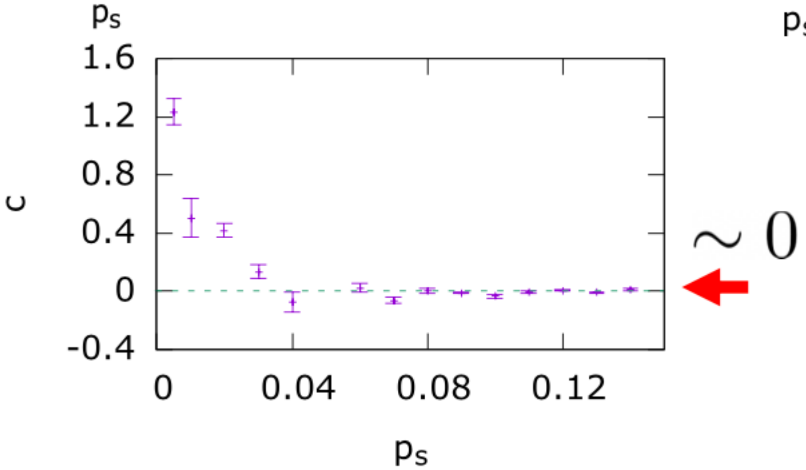


$L_j = n_j$

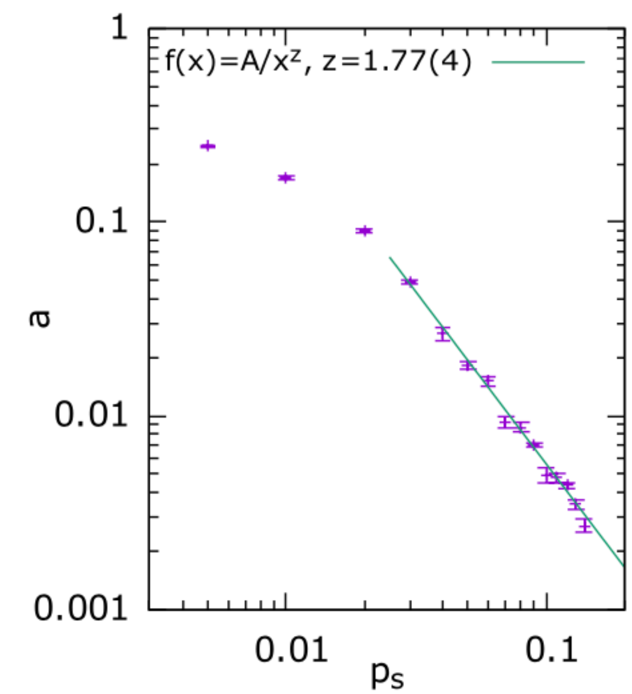
The **volume-law** **is robust:** $f(L) = aL + b + c/L$







No apparent transition to log-law, neither to area-law



$a(p_s) \sim p_s^{-z}, \quad z \approx 1.77$

G. Bandini, G. Piccitto, A. Russomanno, DR, in preparation

Summary

Fate of **entanglement transitions** in *monitored fermionic models*

- qualitatively different behaviors depending on the **specific unraveling** of the Lindblad master equation
- impact of the **range of Lindblad dissipators**
- Impact of the **Hamiltonian interactions**

Area-law vs sub-extensive vs volume-law phase

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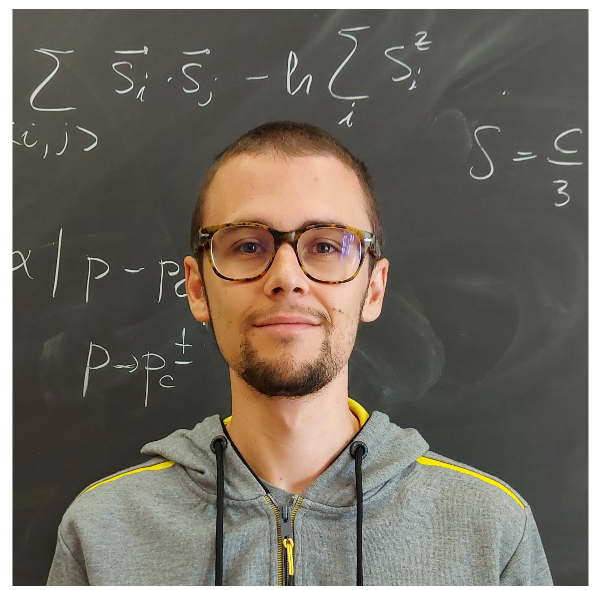

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